

Holographic entanglement entropy of de Sitter braneworld

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We study the holographic representation of the entanglement entropy, recently proposed by Ryu and Takayanagi, in a braneworld context. The holographic entanglement entropy of a de Sitter brane embedded in an anti-de Sitter (AdS) spacetime is evaluated using geometric quantities, and it is compared with two kinds of de Sitter entropy: a quarter of the area of the cosmological horizon on the brane and entropy calculated from the Euclidean path integral. We show that the three entropies coincide with each other in a certain limit. Remarkably, the entropy obtained from the Euclidean path integral is in precise agreement with the holographic entanglement entropy in all dimensions. We also comment on the case of a five-dimensional braneworld model with the Gauss-Bonnet term in the bulk.

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I. INTRODUCTION

The exact description of the de Sitter entropy [1, 2] is still an elusive problem [3, 4]. To clarify the microscopic origin of the de Sitter entropy is surely the first step toward understanding the fundamental nature of spacetime. Since de Sitter spacetime has a cosmological event horizon that has many formal similarities with a black hole event horizon, it is natural to expect that quantum theory of black holes is closely related to that of de Sitter spacetime. Recent remarkable progress of string theory in many non-perturbative phenomena, such as strong/weak duality and the new solitonic objects, D-branes, elucidated the microscopic interpretation of black hole entropy [5]. However, there is severe adversity for discussing de Sitter gravity and the procedure of microstate counting breaks down in string theory. In this paper we address this issue in the braneworld context [6, 7, 8].

Recently Ryu and Takayanagi proposed the holographic derivation for the entanglement entropy [9]. The entanglement entropy emerges due to the existence of an inaccessible region. Consider n -dimensional static spacetime whose $t = \text{const.}$ static slice is divided into two parts by a $(n-2)$ -dimensional surface S_{n-2} . Through the anti-de Sitter/conformal field theory (AdS/CFT) correspondence [10], it is proposed that the entanglement entropy of CFT _{n} for accessible subspace is given by

$$S_{\text{ent}} = \frac{A_{n-1}}{4G_{n+1}}, \quad (1)$$

where G_{n+1} is the gravitational constant in $(n+1)$ -dimensional spacetime and A_{n-1} is the area of a $(n-1)$ -dimensional minimal surface Σ_{n-1} . The minimal surface Σ_{n-1} is a submanifold in AdS_{n+1} whose boundary is given by S_{n-2} . This conjecture means that the entanglement entropy of CFT is given by the a holographic screen in dual AdS in the gravity side, which is called the holographic entanglement entropy. Ryu and Takayanagi confirmed the validity of the proposal for two-dimensional

CFT and thermal CFT when applied to AdS_3 . They also discussed the validity for higher dimensional cases. The proposal is limited to static spacetime since their proposal is motivated by the holographic arguments in black holes [11] and the minimal surface is the horizon in static spacetime, although generalization to the stationary spacetime could be possible.

In the braneworld context, the proposal of the holographic entanglement entropy gives us the motivation for evaluating the entanglement entropy on the brane. Emparan applied their argument to the black hole localized on the brane, and confirmed that the holographic entanglement entropy is identical to the Bekenstein-Hawking entropy on the brane in a certain limit [12]. It is greatly attractive to examine the holographic entanglement entropy for the braneworld black hole because the minimal surface in the bulk is identical to the black hole event horizon. It also illuminates the mimic AdS/CFT correspondence in the Randall-Sundrum braneworld [13, 14], where the bulk gravity dual is the brane gravity coupled with CFT, rather than pure CFT. See Ref. [15] for other applications.

In this paper, we address the relation between the holographic entanglement entropy and the other definitions of entropy of the de Sitter brane. We consider the Z_2 symmetric braneworld model. The problem is reduced to a purely geometrical problem, that is, finding the bulk minimal surface with the fixed boundary which is the cosmological horizon on the de Sitter brane. Although the minimal surface with a fixed boundary is not orthogonal to the brane in general, we can easily show the orthogonality thanks to the Z_2 symmetry across the brane. See Ref. [16] for a similar argument in the lower dimensional braneworld. We also comment on the application to the braneworld model with the Gauss-Bonnet term, which is regarded as a string correction to low energy effective theory.

The rest of this paper is organized as follows. In the next section, we compute the holographic entanglement entropy in the $(n+1)$ -dimensional Randall-Sundrum

braneworld with the n -dimensional de Sitter brane, and show that it agrees with the de Sitter entropy on the brane derived by the Euclidean path integral. Sec. III gives the discussion for the higher derivative theory in five dimensions. In Sec. IV, we summarize our results and discuss the future problems. In Appendix A, we describe the derivation of the minimal surface in the bulk that is needed for the calculation of the holographic entanglement entropy.

II. HOLOGRAPHIC ENTANGLEMENT ENTROPY OF DE SITTER BRANEWORLD

The system we consider here is the n -dimensional de Sitter brane in $(n+1)$ -dimensional AdS spacetime. The $(n+1)$ -dimensional bulk metric is

$$\begin{aligned} ds^2 &= dr^2 + (lH)^2 \sinh^2(r/l) \times \\ &\quad [-dt^2 + H^{-2} \cosh^2(Ht) d\Omega_{n-1}^2] \\ &= dr^2 + (lH)^2 \sinh^2(r/l) \times \\ &\quad \left[-(1 - H^2 \rho^2) dT^2 + \frac{d\rho^2}{(1 - H^2 \rho^2)} + \rho^2 d\Omega_{n-2}^2 \right] \end{aligned} \quad (2)$$

where l is the bulk AdS curvature length and H is the Hubble parameter on the brane located at $r = r_0$. The relation between static coordinates and global coordinates in dS is found in [17]. The cosmological event horizon is located at $\rho = 1/H$. H can be written as

$$H^{-1} = l \sinh(r_0/l). \quad (3)$$

The $(n+1)$ -dimensional gravitational constant is related to the n -dimensional one as [7]

$$G_n = \frac{n-2}{2l} \coth(r_0/l) G_{n+1}. \quad (4)$$

The brane tension σ is related to the AdS curvature scale through Israel's junction condition as

$$\sigma = \frac{(n-1)}{4\pi G_{n+1} l} \coth(r_0/l). \quad (5)$$

Following Ref. [9], the entanglement entropy is given by

$$S_{\text{ent}} := \frac{A_{n-1}}{4G_{n+1}}, \quad (6)$$

where A_{n-1} is the area of the bulk minimal surface on $T = \text{constant}$ static slices. As shown in Appendix A, the bulk minimal surface is the $(n-1)$ -dimensional surface with $\rho = H^{-1}$. Its area is given by

$$A_{n-1} = 2\Omega_{n-2} l^{n-2} \int_0^{r_0} dr \sinh^{n-2}(r/l), \quad (7)$$

where Ω_{n-2} is the area of a $(n-2)$ -dimensional unit sphere, $\Omega_{n-2} = 2\pi^{\frac{n-1}{2}}/\Gamma(\frac{n-1}{2})$. Then the holographic

entanglement entropy is evaluated as

$$S_{\text{ent}} = \frac{(n-2)\Omega_{n-2} l^{n-2}}{4G_n} \coth(r_0/l) \int_0^{r_0/l} dx \sinh^{n-2} x. \quad (8)$$

The de Sitter entropy for n -dimensional de Sitter spacetime is given by [1, 2]

$$S_{\text{area}} = \frac{A_{\text{ch}}}{4G_n} = \frac{\Omega_{n-2}}{4G_n H^{n-2}}. \quad (9)$$

In general S_{ent} is not equal to S_{area} . For $r_0/l \gg 1$, however, the relation between the AdS curvature length and the cosmological horizon radius reduces to $e^{r_0/l} \simeq 2(Hl)^{-1}$ recalling Eq. (3), and so we have

$$S_{\text{ent}} \simeq \frac{\Omega_{n-2} l^{n-2}}{4G_n} \left(\frac{e^{r_0/l}}{2} \right)^{n-2} \simeq \frac{\Omega_{n-2}}{4G_n H^{n-2}} = S_{\text{area}}. \quad (10)$$

Thus the holographic entanglement entropy coincides with the de Sitter entropy on the brane in the limit of $r_0/l \gg 1$. The discrepancy for general r_0 is not surprising because the gravitational field equations on the brane are different from the standard Einstein equations due to the brane/bulk interaction [18], and hence the correction term to the entropy should be taken into account.

The simple way to evaluate the de Sitter entropy on the brane taking such effect into account is to use the Euclidean path integral [2]. If the gravity theory is weakly coupled, the dominant contribution to the path integral is made by those which are an extremum of the action and, then, the partition function can be adequately approximated by the classical action, $Z = e^{I_E}$, where I_E is the Euclidean action. In the braneworld, we stress that n -dimensional computation corresponds to $(n+1)$ -dimensional Euclidean path integral. Because the n -dimensional effective action on the brane is given by the integration of the $(n+1)$ -dimensional action over the coordinate of the extra dimension. Therefore, we should compute the n -dimensional de Sitter entropy by $(n+1)$ -dimensional action. Accordingly, the entropy is given by

$$S_E = \beta \langle E \rangle - I_E = -I_E, \quad (11)$$

where the total energy $\langle E \rangle$ vanishes in the present case. Note that we should take the total action for the partition function because the bulk gravity contributes to the effective theory on the brane as mentioned above. However, because the bulk is static and has no event horizon, the entropy obtained by this partition function surely corresponds to the entropy which stems from the cosmological horizon on the brane.

The total action I_E is evaluated as

$$\begin{aligned}
I_E &= \frac{1}{16\pi G_{n+1}} \int d^{n+1}x \sqrt{g} \left[-\frac{n(n-1)}{l^2} - R \right] \\
&\quad + \int d^n x \sqrt{q} \left[\sigma - \frac{1}{8\pi G_{n+1}} K \right] \\
&= \frac{n}{8\pi G_{n+1} l^2} \int d^{n+1}x \sqrt{g} - \frac{1}{n-1} \sigma \int d^n x \sqrt{q} \\
&= \frac{n-2}{4(n-1)} \frac{l^{n-2}}{G_n} \coth(r_0/l) \left[n \int_0^{r_0/l} dx \sinh^n x \right. \\
&\quad \left. - \coth(r_0/l) \sinh^n(r_0/l) \right] \Omega_{n-2} \\
&= -\frac{(n-2)\Omega_{n-2} l^{n-2}}{4G_n} \coth(r_0/l) \int_0^{r_0/l} dx \sinh^{n-2} x \\
&= -S_{\text{ent}}, \tag{12}
\end{aligned}$$

where g and q are the determinant of Euclidean metric of the bulk and the brane, respectively, and R is the Ricci scalar of the bulk spacetime. We can see that S_E agrees exactly with the holographic entanglement entropy.

It is expected that the difference between S_{area} and $S_{\text{ent}} = S_E$ comes from the higher order corrections, i.e., the Kaluza-Klein corrections to the brane theory. To see this, we focus on the $n = 4$ case. From Eqs. (8) and (9) the difference between S_{ent} and S_{area} is evaluated as

$$\frac{S_{\text{ent}} - S_{\text{area}}}{S_{\text{area}}} \simeq (Hl)^2 \log(Hl/2) + \dots \tag{13}$$

The effective action I_{eff} on the brane is given by [13]

$$\begin{aligned}
I_{\text{eff}} &\simeq \frac{1}{16\pi G_4} \int d^4x \sqrt{-q} \left[{}^{(4)}R + 4l^2 \log(Hl/2) \mathcal{R}_2 \right] \\
&\quad - \int d^4x \sqrt{-q} (\sigma - \sigma_{\text{RS}}) + \Gamma_{\text{CFT}}, \tag{14}
\end{aligned}$$

where $\mathcal{R}_2 := -(1/8){}^{(4)}R_\nu^\mu {}^{(4)}R_\mu^\nu + (1/24)[{}^{(4)}R]^2$ with the Ricci tensor in the bulk ${}^{(4)}R_\nu^\mu$, σ_{RS} is the Randall-Sundrum canonical tension defined by $\sigma_{\text{RS}} := 3/(4\pi G_5 l)$ [8], and Γ_{CFT} is the effective action for CFT on the brane. From the trace part of the effective field equations, we see

$$\begin{aligned}
{}^{(4)}R &= -8\pi G_4 [T_{\text{CFT}} - 4(\sigma - \sigma_{\text{RS}})] \\
&= 2 \left[-\frac{6}{l^2} + \frac{1}{6} (8\pi G_5 \sigma)^2 \right] \\
&= 12H^2, \tag{15}
\end{aligned}$$

where we used the fact that the trace of the effective stress tensor of CFT, T_{CFT} , is given by the trace anomaly and the first line of the above equation is equivalent to the trace part of the effective equations derived by the geometrical projection [18]. Therefore the Euclidean effective action gives us the entropy

$$S_{\text{ent}} \simeq S_{\text{area}} \left[1 + \frac{2}{3} l^2 H^{-2} \log(Hl/2) \mathcal{R}_2 \right]. \tag{16}$$

Since ${}^{(4)}R_{\mu\nu} \simeq 3H^2 q_{\mu\nu}$, we have $\mathcal{R}_2 \simeq \frac{3}{2} H^4$. Thus we finally obtain the same expression as Eq. (13). From the above analysis, we conclude that the discrepancy between the holographic entropy S_{ent} and the de Sitter entropy S_{area} comes from the higher derivative correction to the field equations on the brane.

III. HIGHER CURVATURE TERMS IN THE BULK

Now let us discuss Ryu and Takayanagi's proposal in theory with higher derivative terms in the bulk. Although the low-energy effective action of string theory is the Einstein-Hilbert action, it will receive the corrections at high energies, which will appear as a series of higher derivative terms. For simplicity, we concentrate on the Gauss-Bonnet correction term.

There would be no reason why the current proposal of holographic entanglement entropy holds in this setup, because it is based on the original AdS/CFT duality without higher derivative corrections. Therefore, it is worth discussing the effect of higher derivative terms into the holographic entanglement entropy in the braneworld. See Ref. [19] for related issues from different point of view.

The action with the Gauss-Bonnet term is given by [20]

$$\begin{aligned}
S &= \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[R - 2\Lambda + \frac{\beta l^2}{4} (R^2 - 4R_{AB}R^{AB} \right. \\
&\quad \left. + R_{ABCD}R^{ABCD}) \right] + \int d^4x \sqrt{-q} (-\sigma + Q), \tag{17}
\end{aligned}$$

where

$$Q = 2K + \beta l^2 (J - 2{}^{(4)}G_\nu^\mu K_\mu^\nu), \tag{18}$$

with

$$J = -\frac{2}{3} K_\alpha^\mu K_\beta^\alpha K_\mu^\beta + K K_\beta^\alpha K_\alpha^\beta - \frac{1}{3} K^3. \tag{19}$$

Here R_{ABCD} , ${}^{(4)}G_\nu^\mu$ and K_ν^μ are the five-dimensional Riemann tensor, the four-dimensional Einstein tensor and the extrinsic curvature on the brane, respectively. If the bulk is described by AdS spacetime with the curvature radius l , the bulk gravitational equations imply the following relation:

$$\Lambda = -\frac{6}{l^2} + \frac{3\beta}{l^2}. \tag{20}$$

The junction condition gives us the relation between l and the brane tension σ as

$$\frac{1}{l} \coth(r_0/l) \left[1 - \frac{\beta}{3} + \frac{2\beta}{3 \sinh^2(r_0/l)} \right] = \frac{4\pi G_5}{3} \sigma. \tag{21}$$

Using the above equations, the de Sitter entropy on the brane in this theory can be computed from the Euclidean path integral [21] as

$$\begin{aligned} S_E &= -I_E \\ &= \frac{\pi l^3}{G_5} [(1 + \beta) \sinh(r_0/l) \cosh(r_0/l) \\ &\quad - (1 - 3\beta) r_0/l]. \end{aligned} \quad (22)$$

In the limit of $r_0/l \gg 1$, this reduces to

$$S_E \simeq (1 + \beta) \frac{\pi l}{G_5 H^2}. \quad (23)$$

If the entanglement entropy for the Gauss-Bonnet theory is given by the same formula proposed by Ryu and Takayanagi, we have the same result in the previous subsection because the bulk metric has the identical form:

$$S_{\text{ent}} = \frac{\Omega_2 l^2}{2G_5} \int_0^{r_0} dr \sinh^2(r/l) \simeq \frac{\pi l}{G_5 H^2}. \quad (24)$$

Thus there is a discrepancy by the factor $(1 + \beta)$ between the two entropies¹. This result suggests that the holographic entropy proposal based on the minimal surface is not applicable to higher derivative theories. Although we have observed the discrepancy here, we do not intend to draw a negative conclusion for the geometrical description of the entanglement entropy. In particular, because both S_E and S_{ent} have the same dependence on H and l , the result might indicate that the simple modification of the definition S_{ent} could give the exact value of the de Sitter entropy, although further studies are needed to obtain the definite conclusion.

IV. SUMMARY AND DISCUSSION

In this paper we have studied the holographic entanglement entropy for a de Sitter braneworld embedded in an AdS bulk spacetime. We showed that the holographic entanglement entropy S_{ent} exactly agrees with the de Sitter entropy on the brane S_E evaluated by the Euclidean path integral. On the other hand, S_{ent} is not equal to the entropy S_{area} calculated from the horizon area formula on the brane in general because the effective theory on the brane deviates from the conventional Einstein theory. However, they coincide with each other in the limit of $r_0/l \gg 1$. This result is reasonable because in the limit the brane is sent off to the conformal boundary of AdS and then the brane theory decouples from bulk gravity, reproducing Einstein gravity on the brane.

We have also discussed the coverage of the holographic entanglement entropy for the braneworld model with the Gauss-Bonnet term. The current formulation was found to be not applicable for this corrections, and so the reformulation of the holographic entanglement entropy is awaited.

In this paper, we have concentrated on the de Sitter braneworld for simplicity. We expect, however, that the present discussion can be generalized to the Friedmann-Robertson-Walker (FRW) braneworld. We can compute the holographic entropy once the minimal surface is given, but the problem is that the entanglement entropy on the FRW brane is obscure due to the dynamical nature of the FRW spacetime.

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APPENDIX A: MINIMAL SURFACES

In this section, we will show that $\rho = H^{-1}$ is the minimal surface in the bulk as well as on the brane. We begin with the review of the well-known result that the horizon is the minimal surface on static slices. Hereafter, we simply call the surface with zero expansion rate the horizon.

The future directed null vector n_{\pm}^a of out/in going null geodesics are locally decomposed as $n_{\pm}^a = t^a \pm s^a$, where t^a is the future directed timelike vector and s^a is the unit normal spacelike vector. Then the horizon is located at the zero expansion rate of null geodesic congruences θ_{\pm} in static spacetimes. The expansion rate is written as

$$\theta_{\pm} = {}^{(n)}K - {}^{(n)}K_{ab} s^a s^b \pm {}^{(n-1)}k, \quad (A1)$$

where ${}^{(n)}K_{ab} = (g_a^c + t_a t^c) \nabla_c t_b = h_a^c \nabla_c t_b$ and $k = h^{ab} \nabla_a s_b$. Let t^a be a normal vector to the static slices. With $K_{ab} = 0$, we obtain

$$\theta_{\pm} = \pm {}^{(n-1)}k. \quad (A2)$$

Therefore, $k = 0$ if $\theta_{\pm} = 0$.

The induced metric of $T = \text{constant}$ surface is

$$\begin{aligned} h_{ab} dx^a dx^b &= dr^2 + (lH)^2 \sinh^2(r/l) \times \\ &\quad [(1 - H^2 \rho^2)^{-1} d\rho^2 + \rho^2 d\Omega_{n-2}^2]. \end{aligned} \quad (A3)$$

¹ The relation between G_5 and G_4 is not directly related to our current consideration. It is known that it has the complicated scale dependence. See Ref. [22] for the detail of gravity on a de Sitter brane.

On the brane, the minimal surface is the cosmological horizon at $\rho = H^{-1}$. Our task is to find the *bulk* minimal surface with the boundary at the cosmological horizon on the brane.

Let $\rho = h(r)$ be the orbit of the bulk minimal surface, the unit normal vector to the surface is given by

$$s^a = \frac{1}{\sqrt{h_{,r}^2 + \frac{1-H^2\rho^2}{(lH)^2 \sinh^2(r/l)}}} \times \left[-h_{,r}(\partial_r)^a + \frac{1-H^2\rho^2}{(lH)^2 \sinh^2(r/l)}(\partial_\rho)^a \right]. \quad (\text{A4})$$

From the definition of the minimal surface, the trace of the extrinsic curvature vanishes:

$$\begin{aligned} {}^{(n-1)}k &= {}^{(n)}D_a s^a \\ &= \partial_r s^r + \partial_\rho s^\rho + (\Gamma_{rr}^r + \Gamma_{\rho r}^\rho + \Gamma_{A\rho}^A) s^r \\ &\quad + (\Gamma_{r\rho}^r + \Gamma_{\rho\rho}^\rho + \Gamma_{A\rho}^A) s^\rho = 0, \end{aligned} \quad (\text{A5})$$

where ${}^{(n)}D_a$ is the covariant derivative with respect to h_{ab} and A, B denote the coordinate of $(n-2)$ -dimensional unit sphere.

We first show $s^r|_{\text{brane}} \propto h'|_{\text{brane}} = 0$ which indicates that the bulk minimal surface is orthogonal to the brane. To see this, we take the integration around the neighborhood of the brane along the coordinate of the extra dimension r . Then we obtain

$$\lim_{r \rightarrow r_0-0} s^r = \lim_{r \rightarrow r_0+0} s^r. \quad (\text{A6})$$

Due to the Z_2 symmetry, we see

$$\lim_{r \rightarrow r_0-0} h' = - \lim_{r \rightarrow r_0+0} h' \quad (\text{A7})$$

and this implies

$$\lim_{r \rightarrow r_0-0} s^r = - \lim_{r \rightarrow r_0+0} s^r. \quad (\text{A8})$$

Eqs. (A6) and (A8) yield

$$\lim_{r \rightarrow r_0-0} s^r = \lim_{r \rightarrow r_0+0} s^r = 0. \quad (\text{A9})$$

Usually the bulk minimal surface with the fixed boundary on the brane is not orthogonal to the brane. However, the Z_2 symmetry requires the orthogonality. It is reminiscent of the fact that the bulk minimal surface with the free boundary bounded in the brane is orthogonal to the brane.

Let us take the $(n-1)$ -dimensional surface of $h(r) = \text{constant}$. Then $s^r \propto h_{,r} = 0$ on the surface. Eq. (A5) and the normal vector s^a become

$${}^{(n-1)}k = \partial_\rho s^\rho + (\Gamma_{\rho\rho}^\rho + \Gamma_{A\rho}^A) s^\rho \quad (\text{A10})$$

and

$$s^a = \sqrt{\frac{1-H^2\rho^2}{(lH)^2 \sinh^2(r/l)}} (\partial_\rho)^a. \quad (\text{A11})$$

Using Eq. (A11), Eq. (A10) can be rewritten as

$${}^{(n-1)}k = \Gamma_{A\rho}^A s^\rho = \frac{n-2}{\rho} s^\rho. \quad (\text{A12})$$

Therefore, from the expression of s^ρ , the $\rho = H^{-1}$ surface in the bulk is turned out to be the bulk minimal surface.

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